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RECOGNITION ALGORITHM FOR OBJECTS DESCRIBED BY SETS OF FEATURE MEASUREMENTS

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16. Abstract The authors consider a generalization of a recognition problem discussed in two previous publications to the case when the space of feature measurements for the objects to be discriminated is not a metric space, the classes to which the classified objects must be assigned intersect and partial ordering relations are given for the sets of feature measurements and classes. The authors define classification rules and describe an optimization problem which must be solved to obtain the best recognition algorithm for the given class of objects which reduces to finding a set of parameters that maximize a certain functional described in the previous publications. The authors also obtain the partial ordering relations for the classes when partial ordering relations are only given for the sets of feature measurements and in each class.

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RECOGNITION ALGORITHM FOR OBJECTS DESCRIBED BY SETS OF FEATURE MEASUREMENTS

Yu.I. Zhuravlev, Ye. Karchenski, and M. Mikhalevich

Articles [1, 2] considered the recognition problem for /1*
objects described by a finite set of features 1, 2, ..., n. It was assumed that the set of measurements for each feature i is a quasi-metric space.

This means that for each pair (x, y) in M_i we can define a numerical valued function $\rho(x, y)$ such that $\rho(x, x) = 0$ and $\rho(x, y) > 0$ whenever $x \neq y$. In this article, we will consider objects described by sets of feature measurements for which the set of values is not a quasi-metric space.

It is assumed that for some sets only partial ordering relations are given. Usually the partially ordered sets which are considered are sets of classes or sets of values of the feature measurements.

Here we will consider several cases.

1. Suppose that the objects that must be discriminated are described by sets of values of the feature measurements 1, 2, ..., n. The set M_i of values of the i-th feature is finite and consists of the elements $n_{i1}, n_{i2}, \ldots, n_{ik(i)}$.

It is known that the objects that must be discriminated can be subdivided into classes $K_1, K_2, \ldots, K_l$ which, in general, intersect. For each element n_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots,$
$k(i)$ in the set $\{K_1, K_2, \ldots, K_l\}$ a transitive partial ordering /2
R_{ij} is defined. Generally for different pairs i, j the orderings

*Numbers in the margin indicate pagination in the foreign text.

R_{ij} are different. In our discussion, the set of orderings R_{ij} replaces the learning table T_{nm}.

Suppose we are given a set of orderings R_{ij} and an admissible word S.

$$S = (\alpha_1, \alpha_2, \ldots, \alpha_n),\ \alpha_i \in M_i,\ i = 1, 2, \ldots, n.$$

We must find the classification vector $\tilde{\alpha}(S) = (\alpha^1, \alpha^2, \ldots, \alpha^l)$ [2]. We recall that $(\alpha^j = 1) \sim (S \in K_j)$, $(\alpha^j = 0) \sim (S \bar{\in} K_j)$, $j = 1, 2, \ldots, l$.

The class of recognition algorithms will be described in stages. Some of these are the same as the stages presented in [1]. Others are new.

1. All Features (Values of the Feature Measurements) for Class K_j

We select in row S the value α_i of the i-th feature measurement. The ordering relation R_{iu} corresponds to the value α_i (α_i = $= n_{iu}$). Let us consider the diagram of the finite structure generated by the ordering relation R_{iu} in the set $\{K_1, K_2, \ldots, K_l\}$.

Let us associate with the class K_q (in the fourtuple (K_q, K_j, n_{iu}, i) the elements are classes, the number of the feature, the number of the value), the numerical parameters x^1_{qjiu}, x^2_{qjiu}, x^3_{qjiu}.

We associate with the pairs i, u (i is the number of the feature, u is the number of the value of the feature measurement) the numerical parameters γ_{iu}.

For the class K_j in the ordering relation R_{iu}, we break up the other classes into "+" levels and "-" levels.

In I^+, the level B_1^+ includes all elements which follow immediately the elements of K_j. Suppose that the t - 1st level B_{t-1}^+ with the "+" sign has been determined. The level B_t^+ will /3 include all elements which follow immediately at least one element from level B^+_{t-1}.

Since the number of classes is finite, a finite number of nonempty levels exists.

We include in level B^-_1 all elements which precede immediately K_j. Suppose that we determined the t - 1st level B^-_{t-1} with the "–" sign.

We include in level B^-_t all elements which precede immediately the elements in level B^-_{t-1}.

We associate the parameters β^-_t and β^+_t with the levels B^-_t, B^+_t.

Let us also denote by Q_{jiu} the set of classes which cannot be compared with K_j in the structure R_{iu}.

We let

$$P^j_{iu}(S) = \gamma_{iu}\left(\sum_{t=1}^{\infty}\sum_{K_q\in B^+_t} x^1_{qjiu}\cdot\beta^+_t + \sum_{t=1}^{\infty}\sum_{K_q\in B^-_t} x^2_{qjiu}\cdot\beta^-_t + \sum_{K_q\in Q_{jiu}} x^3_{qjiu}\right).$$

The infinite sums in formula $P^j_{iu}(S)$ are in fact finite sums, since the number of nonempty levels B^-_t, B^+_t in a finite structure is finite.

The formula for $P^j_{iu}(S)$ depends on the parameter γ_{iu} which ensures that all considerations pertain to the u-th element from the range of measurement values of the i-th feature.

The parameter X^1_{qjiu} ensures that the class K_q follows the class K_j in the structure R_{iu}. Also, K_q is at the distance t /4 from K_j in the structural diagram (this is ensured by the parameter β_t^+).

The parameter X_{qjiu}^2 ensures that the class K_q precedes the class K_j in the structure R_{iu}. K_q is at the distance t from the class K_j in the R_{iu} diagram (this is ensured by the parameter β_t^-).

The parameter X_{qjiu}^3 ensures that the classes K_q and K_j cannot be compared in the structure R_{iu}.

All parameters that were introduced are numerical parameters.

2. Recognition Algorithm

Let $\Gamma_j(S) = \frac{1}{M} \sum_{i=1}^{M} P_{iu}^j(S)$. The quantity $\Gamma_j(S)$ will be called the estimate for class K_j, $j = 1, 2, \ldots, \ell$ over the row S.

We introduce the parameters $\delta_{11}, \delta_{12}, \ldots, \delta_{1\ell}, \delta_{21}, \delta_{22}, \ldots, \delta_{2\ell}$, $1 \geq \delta_{ij} \geq 0$. We construct the quasi-classification vector $(\Delta_1, \Delta_2, \ldots, \Delta_\ell)$ for the row S: $\Delta_j = 1$ (S belongs to K_j) if

$$\frac{\Gamma_j(S)}{\sum_{i=1}^{\ell} \Gamma_i(S)} \geq \delta_{1j}, \quad 1 \leq j \leq \ell;$$

$\Delta_j = 0$ (S does not belong to K_j), if

$$\frac{\Gamma_j(S)}{\sum_{i=1}^{\ell} \Gamma_i(S)} \leq \delta_{2j}, \quad 1 \leq j \leq \ell;$$

$$\Delta_j = \text{"} - \text{"}$$

(the algorithm did not make a decision whether S belongs or does not belong to K_j); if

$$\delta_{1j} < \frac{\Gamma_j(S)}{\sum_{i=1}^{\ell} \Gamma_i(S)} < \delta_{2j}, \quad 1 \leq j \leq \ell.$$

As in the previous studies [1, 2], a quality functional $\phi(A)$ can be easily introduced for the algorithm A by examining /5 the learning table T'_{nm}, consisting of the objects whose classification vectors are known. The functional $\phi(A)$ is calculated as follows. For each row S' in T'_{nm}, using the algorithm A, its quasi-classification vector $\tilde{\beta}(S')$ is calculated. The $\tilde{\beta}(S')$ and the classification vector $\tilde{\alpha}(S')$ are compared for all rows in T'_{nm}.

The algorithm parameters are selected in such a way that they determine the algorithm A* which maximizes the functional $\phi(A)$.

3. Extension of the Class of Recognition Algorithms

1. Let us associate with the algorithm A the collection of subsets Ω_A of the set $\langle 1, 2, \ldots, n\rangle$. The elements of Ω_A will be called the reference subsets for algorithm A. In this article, we will consider as Ω_A all subsets of the set $\langle 1, 2, \ldots, n\rangle$ which have exactly k elements.

2. Let $\Omega_k = \langle i_1, i_2, \ldots, i_k\rangle \subseteq \langle 1, 2, \ldots, n\rangle$.
Let us consider $\tilde{\omega} S = (\alpha_{i_1}, \ldots, \alpha_{i_k})$ [1]. We construct according to Section 1.1 the quantities

$$P^j_{i_1 u(i_1)}(S), \ldots, P^j_{i_k u(i_k)}(S), \quad 1 \leqslant i \leqslant l.$$

We introduce the numerical parameters $\varepsilon_1, \ldots, \varepsilon_n$, ε and consider the inequalities $P^j_{i_1 u(i_1)}(S) \geqslant \varepsilon_{i_1}, \ldots, P^j_{i_k u(i_k)}(S) \geqslant \varepsilon_{i_k}$.

We will say that the set of characteristics $\langle i_1, i_2, \ldots, i_k\rangle$ is representative over S for the class K_j if at least $k - \varepsilon$ of the above inequalities are satisfied.

In other words, we say that the representativeness function /6
$\tau^j(\tilde{\omega} S) = 1$. Otherwise, $\tau^j(\tilde{\omega} S) = 0$.

3. Let us introduce the set of numerical parameters $P_{1j}, \ldots, P_{nj}$ (the parameter P_i corresponds to the feature i), $i = 1, 2, \ldots, n$;

$j = 1, 2, \ldots, \ell$.

Let $\Gamma^j(\tilde{\omega} S) = (P_{i_1 j} + \ldots + P_{i_k j}) \cdot \tau^j(\tilde{\omega} S)$.

4. We introduce the estimate $\Gamma_j(S)$ over the row S for class K_j.

$$\Gamma_j(S) = \sum_{\Omega_k \in \Omega_A} \Gamma^j(\tilde{\omega} S)$$

5. We will write out explicitly the estimates $\Gamma_1(S)$, $\Gamma_2(S)$, $\ldots$, $\Gamma_\ell(S)$. The quasi-classification vector for row S in algorithm A is determined in terms of the parameters $\delta_{11}, \ldots, \delta_{1\ell}, \delta_{21}, \ldots, \delta_{2\ell}$ both here and in the algorithms in Sections 1, 2.

The algorithms that were introduced are determined by specifying the parameters which determine the P_{iu} and also by specifying the parameters $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n, \varepsilon, P_{11}, \ldots, P_{n\ell}, \delta_{11}, \ldots, \delta_{1\ell}, \delta_{21}, \ldots, \delta_{2\ell}$. The parameters are determined from the solution of the extremum problem: i.e., a set of parameters is found which determines the best recognition algorithm for the given set of objects [1, 2].

A useful formula can be derived for calculating the quantities $\Gamma_j(S)$, $j = 1, 2, \ldots, \ell$.

Suppose that the quantities $P^j_{1u(1)}, \ldots, P^j_{nu(n)}$ have been calculated for the row S. We fix all numbers $\tau_1, \ldots, \tau_q$ for which the inequalities $P^j_{\tau_i u(\tau_i)} \leq \varepsilon_{\tau_i}$, $i = 1, 2, \ldots, q$ are satisfied.

Let

$$P_{\tau_1 j} + P_{\tau_2 j} + \ldots + P_{\tau_q j} = Q_{qj}$$

$$\sum_{i=1}^{n} P_{ij} - Q_{qj} = Q_{n-qj}.$$

Theorem: /7

$$\Gamma_j(S) = \sum_{t=0}^{\varepsilon} \left(C_q^{k-t-1} \cdot C_{n-q}^{t} \cdot Q_{qj} + C_{n-q}^{t-1} \cdot C_q^{k-t} \cdot Q_{n-qj} \right)$$

Proof

Let us see which sets from k columns make a nonzero contribution to the quantity $\Gamma_j(S)$. Clearly these sets include $k - t$, $0 \leqslant t \leqslant \varepsilon +$ [illegible] columns from those with the numbers τ_1, ..., τ_q and t columns from the remaining n - q columns. Let us consider all such sets for a fixed t.

Each column from the τ_1, ..., τ_q columns is included exactly in $C_q^{k-t-1} \cdot C_{n-q}^{t}$ of the sets that were mentioned. Therefore, the contribution of such a column with the number τ_u is

$$P_{\tau_u j} \cdot C_q^{k-t-1} \cdot C_{n-q}^{t}$$

Each column τ_v among those not included in the set τ_1, ..., τ_q is included exactly in $C_{n-q}^{t-1} \cdot C_q^{k-t}$ sets and its contribution is

$$P_{vj} \cdot C_{n-q}^{t-1} \cdot C_q^{k-t}$$

Summing the contributions of all columns, first for the fixed t, and then over all t, we obtain

$$\Gamma_j(S) = \sum_{t=0}^{\varepsilon} \left(C_q^{k-t-1} \cdot C_{n-q}^{t} \cdot Q_{qj} + C_{n-q}^{t-1} \cdot C_q^{k-t} \cdot Q_{n-qj} \right)$$

which proves the theorem.

II. In many problems, the structures R_{iu} are not given and the partial ordering relations are given for the sets M_i, i = 1, 2, ..., n and each of the classes K_1, ... K_ℓ.

The transitive partial ordering for K_j in M_i will be denoted by $\tilde{R}_{ij}$. /8

It is possible to pass from the set of orderings $\tilde{R}_{ij}$ to the set of orderings R_{iu}. The transition is not one-to-one.

We give two examples of the transition.

1) Suppose that the ordering relations $\tilde{R}_{i\tau}$, $\tilde{R}_{i\rho}$ are given in the set for the classes K_τ, K_ρ, respectively. We will determine in these ordering relations the u-th element of the set. Let $\mathfrak{M}_\tau^+$ be the set of all elements from the set M_i which follow the u-th element in $\tilde{R}_{i\tau}$, $\mathfrak{M}_\tau^-$ the set of all elements which precede the u-th element in $\tilde{R}_{i\tau}$, $\mathfrak{M}_\rho^+$ the set of all elements which follow the u-th element in $\tilde{R}_{i\rho}$, and $\mathfrak{M}_\rho^-$ the set of all elements which precede the u-th element in $\tilde{R}_{i\rho}$.

The ordering relation R_{iu} is constructed according to the rule: $K_\tau \leq K_\rho$ if $\mathfrak{M}_\tau^- \supseteq \mathfrak{M}_\rho^-$ and $\mathfrak{M}_\tau^+ \subseteq \mathfrak{M}_\rho^-$

Clearly the relation that was introduced is transitive.

2) Instead of the sets $\mathfrak{M}_\tau^+, \mathfrak{M}_\tau^-, \mathfrak{M}_\rho^+, \mathfrak{M}_\rho^-$, let us consider their subsets $(\mathfrak{M}_\tau^+)', (\mathfrak{M}_\tau^-)', (\mathfrak{M}_\rho^+)', (\mathfrak{M}_\rho^-)'$ consisting only of those elements which are in an ordered relation with the u-th element of the set M_i in the structures $\tilde{R}_{i\tau}$, $\tilde{R}_{i\rho}$.

The ordering relation R_{iu} will be constructed according to the rule $K_\tau \leq K_\rho$ if $(\mathfrak{M}_\tau^-)' \supseteq (\mathfrak{M}_\rho^-)'$ and $(\mathfrak{M}_\tau^+)' \subseteq (\mathfrak{M}_\rho^+)'$, and the sets $(\mathfrak{M}_\tau^-)', (\mathfrak{M}_\tau^+)'$ are not all empty, which also applies to the sets $(\mathfrak{M}_\rho^-)', (\mathfrak{M}_\rho^+)'$.

In this study we will not dwell in detail on the transition from the ordering relations $\tilde{R}_{ij}$ to the ordering relations R_{iu}. /9

We only mention that it can be selected from the set of all admissible transitions by solving the extremum problem for the selection of the best recognition algorithm.

After the structures R_{iu} have been constructed, the recognition problem in Part II reduces to that in Part I.

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